STRESS DISTRIBUTION IN THE WEDGE-TYPE PISTON AND THE SHAFT OF A SYSTEM FOR THE ADIABATIC COMPRESSION OF A GAS

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In the practice of gasdynamic experimentation, wide use is made of a method for obtaining an adiabatically compressed gas by the braking of a previously accelerated piston. With this process, the required parameters of the compressed gas are usually obtained only in the final stage of the motion, constituting only a small part of the total path of the piston. The recoil of the piston after the compression cycle takes place in very short times, which sharply decreases the time of the existence of high parameters of the gas, and leads to a need to build special valve devices to prevent the back-flow of gas from the forechamber [1]. Such multivalve systems have a low reliability, since, by virtue of the requirements for a small lag and large flow-through cross sections, the number of such valves reaches several tens; in addition, these valves work under conditions of high pressures and temperatures, as well as under the conditions of a strong corrosive action if a chemically active gas is used as a working body.

Basically, the problem of holding the gas can be solved using a wedge-shaped piston, whose construction is described in [2]. Application of this invention in actual practice is bound up with a need to calculate the distribution of the radial stresses in the shaft, which depend both on the geometry of the piston and the maximal pressures, obtained in the unit, as well as on the conditions of the shaft.

The present article describes a method for such a calculation and gives its results.

The wedge-type piston (Fig. 1) consists of a plunger 1, sealing rings 2, and the wedges 3 with plastic rings 4. The piston is accelerated in the shaft 5 by a gas, stored in the tank 6 after the section of the disk 7.

The middle part of the plunger consists geometrically of one or several truncated cones with the half-angle α at the apex, having inverted large bases toward the side of the compressed gas. In the cavity between the shaft and the plunger there are wedges, consisting of sleeves cut along the radius, whose inner surface is conical with the same angle α, and provided with an antifriction coating. The angle α and the angles of slip β_1 and β_2, corresponding to the coefficients of friction at the inner and outer surfaces of the wedge, satisfying the relationships $\beta_2 - \beta_1 > \alpha$ and $\beta_2 > 2\beta_1$. Practically, $\alpha = 5°$, which assures wedging of the piston at the moment of its stopping.

The piston, together with the wedges, is prevented from moving back by the forces of friction at the shaft, constricting the compressed gas. In this case, on the side of the wedges, the shaft is acted on by the radial load, whose distribution along the generatrix forms the subject of the proposed calculation. Solution of this problem makes it possible to find the dependence of the maximal value of the radial stress σ_r on the pressure of the compressed gas and the geometric characteristics of the piston, which permits optimization of its construction.

In accordance with [3], in the case of a stepwise distribution of the radial load on a shaft with an inner radius a and outer radius b, the deviations from the Lamé formulas are insignificant if the axial distance l from the point of the jump is greater than $2\sqrt{b(b-a)}$.

At the leading part of the piston, the jump in the stresses is insignificant, since the contact stresses at the wedges are usually close to the pressure of the gas. A jump in the load at the rear leads to a redistribution of the forces at the trailing part of the piston, but this redistribution can be made small by an appropriate profiling of the shaft or by the installation of a disconnecting flange.

For actuation of the system of wedges it is necessary that the radial dimension of a wedge exceed the difference of the radial deformations of the shaft and the plunger; this dimension can be rather small. This makes it possible to approximate the real stresses by continuous functions and to neglect deformations of the wedges.

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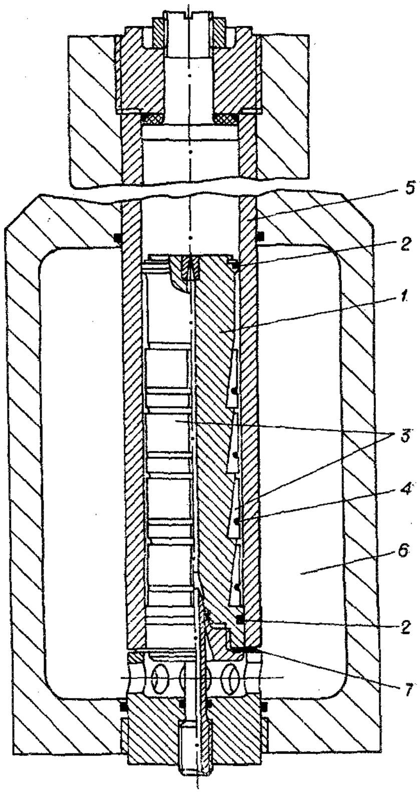

Fig. 1

We shall regard the wedge-type piston as an ordinary plunger, stopping in the shaft at the end of the compression cycle of the gas; under these circumstances, in the stoppered part with a length h, for the stresses the equalities are fulfilled:

$$a^2 d\sigma_p = 2a\,\mathrm{tg}\,\alpha\sigma_r dx = -(b^2 - a^2)d\sigma_s, \tag{1}$$

where $\sigma_s(x)$ and $\sigma_p(x)$ are the fully established distributions of the axial stresses at the shaft and the piston. The total length of the unloaded piston is $L = h + l$.

The relative ε and absolute u radial deformations of the shaft and the piston satisfy the equations [3]

$$\varepsilon_s = \frac{1}{E}\sigma_s + \frac{2a^2\mu}{(b^2 - a^2)E}\sigma_r, \quad \varepsilon_p = \frac{1}{E}\sigma_p - \frac{2\mu}{E}\sigma_r,$$
$$u_s = -\frac{a}{E}\left(\frac{b^2 + a^2}{b^2 - a^2} + \mu\right)\sigma_r - \frac{\mu a}{E}\sigma_s, \quad u_p = \frac{a}{E}(1 - \mu)\sigma_r - \frac{\mu a}{E}\sigma_p. \tag{2}$$

The system is closed by the equation of the compatability of the absolute radial and axial deformations

$$(u_s - u_s^0) - (u_p - u_p^0) = \left[\delta + \int_0^x (\varepsilon_s - \varepsilon_s^0)\,d\xi - \int_0^x (\varepsilon_p - \varepsilon_p^0)\,d\xi\right]\mathrm{tg}\,\alpha. \tag{3}$$

The superscript 0 denotes the parameters of the stressed state, corresponding to the start of the wedging process; δ is the value of the displacement of the rear end of the piston with respect to the shaft after wedging.

Let us determine the form of the functions denoted in Eq. (3) by the superscript 0. If we neglect the value of the pressure of the gas in the tank in comparison with the pressure of the compressed gas, then, at the initial moment of wedging, the axial stresses in the piston in the segment $0 \le x \le h$ are given by a linear dependence (the piston is assumed to be homogeneous over the cross section; the radial dimension of the wedges is neglected)

$$\sigma_p^0(x) = -\frac{p}{L}x. \tag{4}$$

For ε_p^0 and u_p^0 we have

$$\varepsilon_p^0 = -\frac{p}{EL}x, \quad u_p^0 = \frac{\mu a p}{EL}x. \tag{5}$$

The value of the initial axial stresses in the shaft σ_s^0 is uniquely determined, since it depends on the construction of the unit, the conditions of the attachment of the shaft, and is given by the inertial loads in the section of the shaft $0 \le x \le h$ at the moment of the wedging of the piston. The length of the shaft exceeds by at least an order of magnitude the length of the piston, which permits regarding σ_s^0 in the segment $0 \le x \le h$ as an approximately constant quantity.

The mass of the shaft is much greater (by at least two orders of magnitude) than the mass of the piston. If it is assumed that the "infinite" mass is concentrated in the region $x > h$ (the center of mass is ahead of the wedging piston), then, $\sigma_s^0 \approx 0$; if the main mass of the unit is concentrated in the region $x < 0$ (the center of mass is located behind the piston), then $\sigma_s^0 \approx \frac{a^2}{b^2 - a^2}p$.

In real cases, there are different values of the initial stress in the shaft within the limits $0 \leqslant \sigma_s^0 \leqslant \frac{a^2}{b^2 - a^2}p$.

After the introduction of dimensionless parameters and constants, differentiation of Eq. (3) and substitution of (1), (2) into it, taking account of (4), (5), leads to an inhomogeneous linear differential equation

$$\frac{d^2\pi_p}{d\xi^2} - \gamma^2\pi = \frac{\gamma^2(1-\beta^2)}{\lambda}(\mu + \pi_s^0\lambda + \gamma\xi)$$

with the boundary conditions $\pi_p(0) = 0$, $\pi_p(1) = -1$, where

$$\xi = x/h; \quad \pi_p = \sigma_p/p; \quad \gamma = \frac{h \operatorname{tg} \alpha}{a}; \quad \lambda = \frac{L \operatorname{tg} \alpha}{a}; \quad \beta = \frac{a}{b}; \quad \pi_s^0 = \frac{\sigma_s^0}{p}.$$

The solution of this equation is the function

$$\pi_p(\xi) = (1-\beta^2)\left\{\left(\frac{\mu}{\lambda} + \pi_s^0\right)\left[\frac{\operatorname{sh}\gamma\xi + \operatorname{sh}\gamma(1-\xi)}{\operatorname{sh}\gamma} - 1\right] + \frac{\gamma}{\lambda}\left[\frac{\operatorname{sh}\gamma\xi}{\operatorname{sh}\gamma} - \xi\right]\right\} - \frac{\operatorname{sh}\gamma\xi}{\operatorname{sh}\gamma},$$

after whose substitution into (1) we obtain analogous formulas for the distributions of the dimensionless radial ($\pi_r = \sigma_r/p$) and axial ($\pi_s = \sigma_s/p$) stresses in the shaft

$$\pi_r(\xi) = \frac{1-\beta^2}{2}\left[\left(\frac{\mu}{\lambda} + \pi_s^0\right)\frac{\operatorname{ch}\gamma\xi - \operatorname{ch}\gamma(1-\xi)}{\operatorname{sh}\gamma} + \frac{\gamma}{\lambda}\left(\frac{\operatorname{ch}\gamma\xi}{\operatorname{sh}\gamma} - \frac{1}{\gamma}\right)\right] - \frac{\operatorname{ch}\gamma\xi}{2\operatorname{sh}\gamma},$$

$$\pi_s(\xi) = \beta^2\left\{\left(\frac{\mu}{\lambda} + \pi_s^0\right)\left[1 - \frac{\operatorname{sh}\gamma\xi + \operatorname{sh}\gamma(1-\xi)}{\operatorname{sh}\gamma}\right] - \frac{\gamma}{\lambda}\left(\frac{\operatorname{sh}\gamma\xi}{\operatorname{sh}\gamma} - \xi\right)\right\} + \frac{\beta^2}{1-\beta^2}\frac{\operatorname{sh}\gamma\xi}{\operatorname{sh}\gamma}.$$

For the further analysis, the distribution of the radial stresses $\pi_r(\xi)$ is of great interest. In the interval under consideration $0 \le \xi \le 1$, with real values of the constants and a value of π_s^0 between zero and $\beta^2/(1-\beta^2)$ in the whole extension of the wedging section $\pi_r(\xi) < 0$, the maximal values of the radial stresses, depending on the initial conditions, are attained in one of the two, or at both ends of the wedging section, where $\pi_r(\xi)$ can be calculated using the formulas

$$\pi_r(1) = \frac{1-\beta^2}{2}\left\{\left(\frac{\mu}{\lambda} + \pi_s^0\right)\frac{\operatorname{ch}\gamma - 1}{\operatorname{sh}\gamma} + \frac{\gamma}{\lambda}\left(\frac{\operatorname{ch}\gamma}{\operatorname{sh}\gamma} - \frac{1}{\gamma}\right)\right\} - \frac{\operatorname{ch}\gamma}{\operatorname{sh}\gamma},$$

$$\pi_r(0) = \frac{1-\beta^2}{2}\left\{\left(\frac{\mu}{\lambda} + \pi_s^0\right)\frac{1 - \operatorname{ch}\gamma}{\operatorname{sh}\gamma} + \frac{\gamma}{\lambda}\left(\frac{1}{\operatorname{sh}\gamma} - \frac{1}{\gamma}\right)\right\} - \frac{1}{\operatorname{sh}\gamma}.$$

With an increase in the length of the piston and a constant pressure of the constricted gas, the value of the radial stresses should decrease, since the force holding the piston is distributed in the greatest length of the wedging section. As a calculation shows, however, the value of the maximal stresses does not become vanishingly small, but, as a result of the redistribution of the load, is concentrated, depending on the initial conditions, at one or both ends of the piston in the form of peaks which become narrower and narrower, whose value with $\gamma \to \infty$ decreases and, at the limit, is determined from the relationships

$$\pi_r(1) = \frac{1-\beta^2}{2}(\pi_s^0 + 1) - \frac{1}{2}, \quad \pi_r(0) = -\frac{1-\beta^2}{2}\pi_s^0.$$

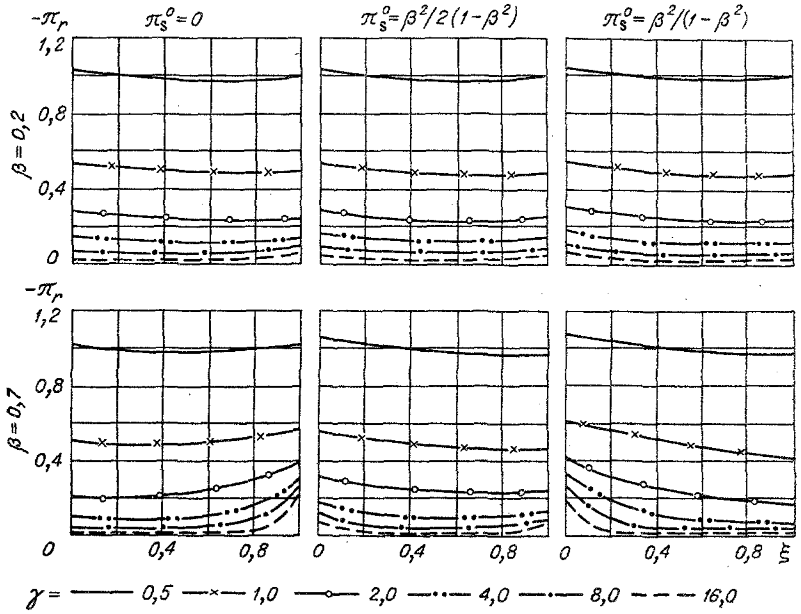

Fig. 2

It can be seen that the first of these values is equal to $-\beta^2/2$ with location of the center of mass ahead of the motion of the piston ($\pi_s^0 = 0$), and to zero with a location behind the piston $\left(\pi_s^0 = \frac{\beta^2}{1-\beta^2}\right)$, the second, on the contrary, is equal to zero in the first case, and to $-\beta^2/2$ in the second case.

The maximal radial stress can be decreased by an appropriate position of the center of mass. For this, it is obviously necessary to satisfy the condition $\pi_r(0) = \pi_r(1)$, using which we can find the required initial stresses in the shaft using the formula

$$\pi_s^0 = \frac{\lambda - \gamma(1-\beta^2) - 2\mu(1-\beta^2)}{2\lambda(1-\beta^2)} = \frac{1}{2(1-\beta^2)} - \frac{\gamma}{2\lambda} - \frac{\mu}{\lambda}$$

or, with large values of γ

$$\pi_s^0 = \frac{\beta^2}{2(1-\beta^2)},$$

which corresponds to the center of mass, located in the middle of the wedging section. In this case, the maximal stresses will be lowered by twice in comparison with a location ahead of or behind this section.

Figure 2 illustrates the dependences $\pi_r(\xi)$ for $\gamma = 0.5$, 1, 2,4, 8, 16; $\beta = 0.2$ and 0.7; $\pi_s^0 = 0$; $\beta^2/2(1-\beta^2)$, $\beta^2/(1-\beta^2)$.

Previous model investigations of the efficiency of the calculating system were made in a device (h = L = 11.5 cm, $\alpha = 2$ cm, b = 10 cm, $\alpha = 5°$), a scheme of which is shown in Fig. 1.

The dimensionless parameters λ, γ, and β for the above dimensions were equal, respectively, to 0.5, 0.5, and 0.2. The value of γ was specially selected in such a way that the radial stresses would have large values, to verify the possibility of holding the piston with a small length. A calculation using the above formulas gives a maximal value of the stresses $\sigma_r(\xi) = 1.02$, i.e., approximately equal to the pressure of the compressed gas.

It must be recalled that the calculation was made for the moment when the pistion is completely stopped and has found an equilibrium position. It is obvious that during the wedging process, there is a certain "settling" of the piston; therefore, depending on the volume of the compressed volume, the pressure of the gas at the end of the wedging process is decreased to one degree or another. Tests of the system in the above-described model unit were made with values of the peak pressures of 1500, 4000, and 8000 atm. The pressure of the gas at the end of the wedging was, respectively, around 1250, 2200, and 3700 atm. Several tens of cycles were carried out for each pressure; the system worked without breakdowns. With work at a maximal pressure, impressions arose at the inner surface of the shaft, i.e., traces of the pressure of the wedges. The identity (visual) of the impressions bears witness to the uniformity of the distribution of the stresses.

In another model, a piston with a diameter of 50 mm was used, having values of λ, γ, and β equal approximately to 0.9, 0.87, and 0.45, respectively. The working pressure of the compressed gas was around 2000 atm. The maximal radial stresses in this case, according to the calculation, around 1200 atm.

It must be noted that, in several hundreds of cycles of work, there was no case of breakdown of the system or of damage to its elements.

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ANALYSIS OF STRUCTURE ELEMENTS TAKING ACCOUNT OF MATERIAL DAMAGE DURING CREEP

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Deformations accumulated in the third stage of creep [1, 2] are neglected in the analysis of structure elements in the majority of cases. However, as follows from an analysis of experimental results [3], some structural materials disclose quite definite third sections of creep even for insignificant deformations on the order of 1-2%.

A standard computation of the stress–strain state for this scheme and the strength analysis of the structure elements under creep conditions do not take account of the fact [1] that cumulative damage, which exerts substantial influence on the creep rate and results in redistribution of the stress field, precedes fracture.

An analysis of vessels stressed by internal pressure is presented below, in which the circumstances noted above are taken entirely into account. The stress–strain state of the vessels and the lower boundary of the fracture time are determined. It is noted that the elucidated method of solution is simpler and more effective in the volume and complexity of the calculational procedures than the traditional methods [1].

Let a uniformly heated vessel (sphere, cylinder) be loaded by a constant internal pressure p with respect to time. The equilibrium equations and boundary conditions have the form [1, 2]

$$\partial\sigma_r/\partial r + k(\sigma_r - \sigma_\varphi)/r = 0,\ a \leqslant r \leqslant b; \tag{1}$$

$$\sigma_r(a) = -p,\ \sigma_r(b) = 0, \tag{2}$$

where a and b are the inner and outer radii, respectively. For a cylindrical vessel k=1, while k=2 for a spherical vessel, and σ_r, σ_φ are the principal stress tensor components, which are functions of the time and the coordinate r. The remaining principal stress σ_θ equals σ_φ [2] for a spherical vessel in the case of central symmetry, and σ_z for a cylindrical vessel is determined from the standard assumption about no creep in the axial direction [1, 2].

The creep strain rate tensor components are related to the displacement velocity vector components by the known Cauchy relations [2], while the equation of continuity of the creep strain rate has the form [2]

$$\partial\eta_\varphi/\partial r + (\eta_\varphi - \eta_r)/r = 0. \tag{3}$$

We write the system of equations describing all three stages of material creep and taking account of the damage process in time in the form [1, 4]

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